# PDESOL EXAMPLE PROBLEMS

Listed below are the sample problem files. These files contain the problem statements and solutions. When you open a sample file, you can read both the problem statement and the solution, or skip the latter. In either case, you can rerun the problems and generate new solutions with the same or different general parameters.

## List of Example Problems

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burgers.pde   - One-dimensional Burgers' Equation                      [48  sec]
CO2.pde       - Seven-Reservoir World Carbon Dioxide model             [1.5 sec]
coolint.pde   - Cooldown of a Superconducting Magnet                   [0.1 sec]
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kdv.pde       - Korteweg-de Vries Equation                             [170 sec]
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```

[The numbers in brackets are approximate run-times obtained on a Pentium 166 MHz machine with 32 MB RAM.]

## Brief Descriptions of Sample Problems

The sample problems are briefly described below. The problem statements include explanatory comments and notes; many of the features illustrated by these problems are also discussed in detail in the on-line help/tutorial file.

### advec.pde: Advection Problem

This is an example of a first-order convective (hyperbolic) PDE. The equation solved is the advection equation:

```
Ut + v Ux = 0
```

The solutions obtained show the propagation of a finite discontinuity (step change) along x. Two equations are solved simultaneously to illustrate the differences between centered and upwind approximations for the convective (dU/dx) terms.

### b&s.pde: Black-Scholes Model for an American Put Option

This example involves derivative security pricing using the Black-Scholes model given by:

```
Ut = -rx Ux - ½ (sx)^2 Uxx + r U
```

Specifically, it is used to value a 5-month American put option on a non-dividend paying stock. The Black-Scholes equation is solved starting at zero time to maturity and marching to "present" (5 months to maturity).

### burgers.pde: One-dimensional Burgers' Equation

This is a standard test problem for PDE numerical methods, with known analytical solutions. The equation is of the hyperbolic-parabolic type:

```
Ut = -U Ux + vis Uxx
```

For small values of 'vis', the Burgers' equation is strongly hyperbolic, and the solutions can exhibit steep moving fronts which are difficult to resolve numerically. This example presents solutions for a case of Burgers' equation (vis = 0.003) with front sharpening as time progresses.

## CO2.pde: Seven-Reservoir World Carbon Dioxide Model

This model treats the carbon dioxide distribution problem in terms of up to seven well-mixed reservoirs. The modified EULER scheme is used to integrate the ODE system.

## coolint.pde: Cooldown of a Superconducting Magnet

This example illustrates the use of tabular input. The specific heat of iron is given in tabular form as a function of temperature.

## davidnko.pde: Davidenko's Method for Nonlinear Algebraic Equations

This illustrates application of an ODE integrator to solve nonlinear algebraic and transcendental equations. Davidenko's method, which is a differential form of Newton's method, is used to solve a 2x2 nonlinear algebraic system.

## fourierc.pde: Fourier Second Law in Cylindrical Coordinates

This is an example of a parabolic PDE of the form:

```
Tt = Trr + (1/r) Tr
```

governing diffusion in cylindrical geometry. For a solid cylinder, the term `"(1/r) Tr"` is indeterminate at r = 0. l'Hospital's rule is used to evaluate this indeterminate form. The problem statement shows how the "divide by zero" at r = 0 can be avoided in PDESOL (through the use of the "greater than" ("x > 0") construct) and how the proper value for the time derivative is specified at r = 0.

## glucose.pde: Pancreatic Response to Glucose Infusion

This considers a mathematical model for the response of the human pancreas to an infusion of glucose (glucose tolerance test). Input variables can be varied to observe responses ranging from hypoglycemic to hyperglycemic. The Step function and the ">" operator are used to express ODEs for the different application regions of the dependent variables.

## humidcol.pde: PI Control of a Packed Humidification Column

This is a dynamic simulation of a packed humidification column with a PI controller. The integral controller equation is replaced by a differential equation for the error integral, resulting a mixed ODE/PDE system. Constraints on the variables are effected through "<" and ">" operators.

## kdv.pde: Korteweg-de Vries Equation

This is a classical nonlinear PDE of the form

```
Ut + 6 U Ux + Uxxx = 0
```

which balances front sharpening and dispersion to produce solitons, i.e. traveling waves that do no change shape or speed. It was originally formulated to model shallow water flow. A special third-order derivative routine is used to evaluate the dispersion term (`Uxxx`) without the need for boundary conditions, as long as the computed solitons do not closely approach the finite boundaries that are used in place of the infinite boundaries.

## kdvpuls.pde: Korteweg-de Vries Equation with Two-Pulse Initial Condition

This is the same problem as above, but with a two-pulse initial condition. The pulses have different amplitudes and speeds. As the solution progresses, the faster pulse catches up and merges with the slower pulse. The two original pulses eventually reappear, and continue to travel in their original shape.

## multireg.pde: Diffusion in Two Regions

This is an example of how PDESOL can be applied solve a system of PDEs defined over different spatial regions or domains. Such problems arise frequently in scientific and engineering applications. This example considers diffusion of a concentration of a substance in two regions; diffusion can take place across the interface between the two regions. One of the regions is mapped onto the other in order to solve the system of equations over the same spatial domain using PDESOL.

## numerint.pde: Numerical Integration Example

This is another unconventional application of an ODE integrator to evaluate one-dimensional integrals. In this example, the error function integral is evaluated by integrating an equivalent ODE with an initial condition.

## river.pde: River Pollution Model w/Point Source

This is an example of a hyperbolic-parabolic or convective-diffusion equation. The equation models pollutant concentrations in a river and includes a spatial point source, turned on during a finite time interval over the course of the solution.

## schrodgr.pde: Cubic Schrodinger Equation

The Cubic Shrodinger Equation (CSE) governs the movement of solitons traveling with constant velocity and amplitude (without changing shape). When separated into real and imaginary parts, the CSE gives two coupled PDEs of the form:

```
Vt + Wxx + (V^2 + W^2) W = 0
Wt - Wxx - (V^2 + W^2) V = 0
```

with `|U| = |V + i W| = sqrt(V^2 + W^2).`

The equations require a fine grid in space in order to resolve the sharp spatial variations of the solitons; 401 points are used in the example (802 equations).

## steepdes.pde: Steepest Descent Method for Nonlinear Algebraic Equations

This is an ODE-based steepest descent method to solve nonlinear algebraic and transcendental equations. It has a smaller domain of convergence than Davidenko's Method, but can compute solutions when the system of nonlinear equations is singular or ill-conditioned. It is applied to the same 2x2 nonlinear system used in the davidnko.pde demo problem, starting with a set of initial conditions for which the Jacobian is singular.

## stiffsys.pde: A Very Stiff ODE System Example

This example considers a second-order, linear ODE system, with a ratio of eigenvalues of 1000000 (L1 = -1000000, L2 = -1), illustrating the use of an explicit integrator (RKF45) and an implicit stiff integrator (LSODES) for the different time scales of the problem. [See the help file for a detailed discussion on Stability and Stiffness].

## wave.pde: One-dimensional Wave Equation

Second order hyperbolic PDEs, i.e. PDEs which are second-order in time, can be integrated by expressing them as systems of two first-order PDEs. The one-dimensional wave equation

```
Utt - Uxx = 0
```

is used as an example to illustrate this procedure.

## 4thordr1.pde & 4thordr2.pde: Fourth-order PDEs

This is extension of the wave equation to fourth-order in space and second order in time:

```
Utt - Uxxxx = 0
```

The example illustrates the use of stagewise differentiation to evaluate fourth-order spatial derivatives. 4thorder1.pde involves Dirichlet boundary conditions and 4thorder2.pde involves Neumann boundary conditions. This second order initial value problem in time is handled by expressing it as a set of two first order equations.